Differential Equations

Case Study Based Questions

Case Study 1

COVID-19 vaccine are delivered to 90 K senior citizens in a state. The rate at which COVID-19 vaccine are given is directly proportional to the number of senior citizens who have not been administered the vaccines. By the end of 3rd week, $\frac{3}{4}$ th number of senior citizens have been given the COVID-19 vaccines. How many will have been given the vaccines by the end of 4th week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(90 - y)$, where x denotes the number of weeks and y the number of senior citizens who have been given the vaccines.

Based on the above information, solve the following questions:

Q1. The order and degree of the given differential equation are:

a. 1 and 1
b. 2 and not defined
c. 1 and 0
d. 0 and 1

Q 2. Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(90 - y)$?

a. Variable separable method

b. Solving homogeneous differential equation

c. Solving linear differential equation

d. All of the above

Q 3. The general solution of the differential equation $\frac{dy}{dx} = k(90 - y)$ is given by:

a. $\log|50 - y| = kx + C$
b. $-\log|90 - y| = kx + C$
c. $\log|90 - y| = \log|kx| + C$
d. $50 - y = kx + C$

Q 4. The value of C in the particular solution given that $y(0) = 10$ and $k = 0.025$ is:

a. $\log 90$ b. $\log \frac{1}{80}$

c. $\log 80$ d. 80

Q 5. Which of the following solutions may be used to find the number of senior citizens who have been given COVID-19 vaccines?

a. $y = 90 - e^{kx}$ b. $y = 90 - e^{-kx}$

c. $y = 90(1 - e^{-kx})$ d. $y = 90(e^{-kx} - 1)$

Solutions

1. Given differential equation is:

$$\frac{dy}{dx} = k(90 - y)$$

Order of equation = Order of the highest order derivative in the given differential equation = 1

and degree of equation = Degree of highest order derivative in the given differential equation = 1

So, option (a) is correct.

2. Given differential equation is:

$$\frac{dy}{dx} = k(90 - y)$$

$$\Rightarrow \quad \frac{dy}{90 - y} = k \cdot dx \quad \ldots(1)$$

Here, we can solve the above equation by variable separable method.

So, option (a) is correct.

3. From eq. (1), $\frac{dy}{90 - y} = k\,dx$

On integrating, we get

$$\int \frac{dy}{90-y} = k\int 1dx$$

$\Rightarrow \qquad -\log|90-y| = kx + C \qquad \ldots(2)$

which is the required general solution.

So, option (b) is correct.

4. Now, put $y(0) = 10$ and $k = 0.025$ in eq. (2), we get

$$-\log|90-10| = k \times 0 + C$$

$\Rightarrow \qquad C = -\log 80 = \log(80)^{-1}$

$\Rightarrow \qquad C = \log\frac{1}{80}$

So, option (b) is correct.

5. Let $y = 90(1-e^{-kx})$ be the solution of the given differential equation.

$\therefore \qquad \text{LHS} = \frac{dy}{dx} = \frac{d}{dx}\{90(1-e^{-kx})\}$

$$= 90(0 + ke^{-kx}) = 90ke^{-kx}$$

Now, $\qquad \text{RHS} = k(90-y)$

$$= k\{90 - 90(1-e^{-kx})\}$$

$$= k\{90 - 90 + 90e^{-kx}\}$$

$$= 90ke^{-kx} = \text{LHS}$$

Thus, $y = 90(1-e^{-kx})$ is the required solution.

So, option (c) is correct.

Case Study 2

In a college hostel accommodating 500 students, one of the hostellers came in carrying Corona Virus and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 100 infected students after 5 days.

Based on the given information, solve the following questions:

Q 1. If $n(t)$ denotes the number of students infected by Corona Virus at any time t, then maximum value of $n(t)$ is:

a. 50 b. 100 c. 250 d. 500

Q 2. $\frac{dn}{dt}$ is proportional to:

a. $n(1000-n)$ b. $n(500-n)$

c. $n(250-n)$ d. $n(500+n)$

Q 3. The value of $n(5)$ is:

a. 1 b. 50 c. 100 d. 500

Q 4. The most general solution of differential equation formed in given situation is:

a. $\frac{1}{500}\log\left|\frac{500-n}{n}\right|=\lambda t+C$

b. $\log\left|\frac{n}{100-n}\right|=\lambda t+C$

c. $\frac{1}{500}\log\left|\frac{n}{500-n}\right|=\lambda t+C$

d. $\log\left|\frac{100-n}{n}\right|=\lambda t+C$

Q 5. The value of n at any time is given by:

a. $n(t)=\frac{500}{1+499\,e^{-500\lambda t}}$ b. $n(t)=\frac{500}{1-499\,e^{-500\lambda t}}$

c. $n(t)=\frac{50}{1-499\,e^{-500\lambda t}}$ d. $n(t)=\frac{50}{499+e^{-500\lambda t}}$

Solutions

1. Since, maximum number of students in hostel is 500.

$\therefore$ Maximum value of $n(t)$ is 500.

So, option (d) is correct.

2. Let n be the number of infected students.

So, $(500-n)$ be the remaining students.

Clearly, according to given information,

$$\frac{dn}{dt} = \lambda n(500-n),$$

where λ is constant of proportionality.

So, option (b) is correct.

3. Since, 100 students are infected after 5 days.

$$\therefore \qquad n(5) = 100$$

So, option (c) is correct.

4. We have, $\frac{dn}{dt} = \lambda n(500-n)$

$$\Rightarrow \int \frac{dn}{n(500-n)} = \lambda \int dt$$

$$\Rightarrow \frac{1}{500}\int\left(\frac{1}{500-n}+\frac{1}{n}\right)dn = \lambda\int dt$$

$$\Rightarrow \frac{1}{500}\left[\frac{\log|500-n|}{-1}+\log|n|\right] = \lambda t + C$$

$$\Rightarrow \frac{1}{500}\log\left|\frac{n}{500-n}\right| = \lambda t + C$$

So, option (c) is correct.

5. When $t = 0$ and $n = 1$,

$$\frac{1}{500}\log\left(\frac{1}{499}\right) = C$$

$$\therefore \frac{1}{500}\left[\log\left|\frac{n}{500-n}\right| - \log\left(\frac{1}{499}\right)\right] = \lambda t$$

$$\Rightarrow \log\left|\frac{499n}{500-n}\right| = 500\lambda t$$

$$\Rightarrow \quad \frac{499n}{500-n} = e^{500\lambda t}$$

$$\Rightarrow \quad n(t) = \frac{500e^{500\lambda t}}{499 + e^{500\lambda t}} = \frac{500}{1 + 499e^{-500\lambda t}}$$

So, option (a) is correct.

Case Study 3

In a murder investigation, a dead body was found by a detective at exactly 9 pm. Being alert, the detective measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F, where the room temperature is 50°F. Also, it is given the body temperature at the time of death was normal, i.e., 98.6°F.

Let T be the temperature of the body at any time t and initial time is taken to be 9 pm.

Based on the above information, solve the following questions:

Q 1. By Newton's law of cooling, $\frac{dT}{dt}$ is proportional to:

a. $T-60$ b. $T-50$ c. $T-70$ d. $T-98.6$

Q 2. When $t=0$, then body temperature is equal to:

a. 50°F b. 60°F c. 70°F d. 98.6°F

Q 3. When $t=2$, then body temperature is equal to:

a. 50°F b. 60°F c. 70°F d. 98.6°F

Q 4. The value of T at any time t is:

a. $50+20\left(\frac{1}{2}\right)^{t}$ b. $50+20\left(\frac{1}{2}\right)^{t-1}$

c. $50+20\left(\frac{1}{2}\right)^{t/2}$ d. None of these

Q 5. If it is given that $\log_e(2.43) = 0.88789$ and $\log_e(0.5) = -0.69315$, then the time at which the murder occur is:

a. 7 : 30 pm
b. 6 : 30 pm
c. 6 : 00 pm
d. 5 : 00 pm

Solutions

1. Given, T is the temperature of the body at any time t.

Then, by Newton's law of cooling, we get $\frac{dT}{dt} = k(T-50)$, where k is the constant of proportionality.

So, option (b) is correct.

2. From given information, we have at 9 pm, temperature is 70°F.

$\therefore$ At $t = 0$,

$T = 70°F$

So, option (c) is correct.

3. From given information, we have

At 11 pm, temperature is 60°F.

$\therefore$ At $t = 2$,

$T = 60°F$

So, option (b) is correct.

4. We have, $\frac{dT}{dt} = k(T-50) \quad \Rightarrow \quad \frac{dT}{T-50} = k\,dt$

On integrating both sides, we get

$$\log|T-50| = kt + \log C$$

$$\Rightarrow \quad T - 50 = Ce^{kt}$$

Clearly, for $t = 0$, $T = 70°$

$$\Rightarrow \quad C = 20°$$

Thus, $T - 50 = 20e^{kt}$

For $t = 2, T = 60°$

$\Rightarrow$ $10 = 20\,e^{2k}$

$\Rightarrow$ $2k = \log\left(\frac{1}{2}\right)$

$\Rightarrow$ $k = \frac{1}{2}\log\left(\frac{1}{2}\right)$

Hence, $T = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$

So, option (c) is correct.

5. We have, $T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$

Now, $98.6 = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$ $[\because T = 98.6°F]$

$\Rightarrow$ $\frac{48.6}{20} = \left(\frac{1}{2}\right)^{\frac{t}{2}}$

$\Rightarrow$ $\log\left(\frac{48.6}{20}\right) = \frac{t}{2}\log\left(\frac{1}{2}\right)$

$\Rightarrow$ $\frac{t}{2} = \frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)}$

$\Rightarrow$ $t = 2\left[\frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)}\right]$

$= 2\left[\frac{\log(2.43)}{\log(0.5)}\right] = 2\left[\frac{0.88789}{-0.69315}\right]$

≈ -2.56

So, it appears that the person was murdered 2.5 h before 9 pm i.e., about 6:30 pm.

So, option (b) is correct.

Case Study 4

A differential equation is said to be in the variable separable form if it is expressible in the form $f(x)\,dx = g(y)\,dy$. The solution of this equation is given by $\int f(x)\,dx = \int g(y)\,dy + C$, where C is the constant of integration.

Based on the above information, solve the following questions:

Q 1. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of '*a*' is:

a. 2 b. -2 c. 3 d. -4

Q 2. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circle with:

a. variable radii and fixed centre (0, 1)
b. variable radii and fixed centre (0, –1)
c. fixed radius 1 and variable centre on *X*-axis
d. fixed radius 1 and variable centre on *Y*-axis

Q 3. If $\frac{dy}{dx} = y+1, y(0) = 1$, then $y(\ln 2)$ is equal to:

a. 1 b. 2
c. 3 d. 4

Q 4. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is:

a. $e^x = \frac{y^3}{3} + e^y + C$ b. $e^y = \frac{x^2}{3} + e^x + C$
c. $e^y = \frac{x^3}{3} + e^x + C$ d. None of these

Q 5. If $\frac{dy}{dx} = y \sin 2x$, $y(0) = 1$, then its solution is:

a. $y = e^{\sin^2 x}$

b. $y = \sin^2 x$

c. $y = \cos^2 x$

d. $y = e^{\cos^2 x}$

Solutions

1. We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$\Rightarrow \quad (ax+3)\,dx = (2y+f)\,dy$

$\Rightarrow \quad a\frac{x^2}{2} + 3x = y^2 + fy + C$ [on integrating]

$\Rightarrow \quad -\frac{a}{2}x^2 + y^2 - 3x + fy + C = 0$

This will represent a circle, if $\frac{-a}{2} = 1 \Rightarrow a = -2$

[$\because$ in circle, coefficient of x^2 = coefficient of y^2]

So, option (b) is correct.

2. We have, $\frac{y\,dy}{\sqrt{1-y^2}} = dx$

On integration, we get $-\sqrt{1-y^2} = x + C$

$\Rightarrow \quad 1 - y^2 = (x+C)^2$

$\Rightarrow \quad (x+C)^2 + y^2 = 1,$

which represents a circle with radius 1 and centre on the *X*-axis.

So, option (c) is correct.

3. $y' = y + 1 \Rightarrow \frac{dy}{y+1} = dx$

$\Rightarrow \quad \ln|y+1| = x + C$ [on integrating]

Now, $\quad y(0) = 1 \quad \Rightarrow \quad C = \ln 2$

$\therefore \quad \ln\left|\frac{y+1}{2}\right| = x \Rightarrow y + 1 = 2e^x$

So, $\quad y(\ln 2) = -1 + 2e^{\ln 2} = -1 + 4 = 3$

So, option (c) is correct.

4. From the given differential equation, we have

$$\frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

$$\Rightarrow \quad e^y dy = (e^x + x^2)\, dx$$

On integrating, we get $e^y = e^x + \frac{x^3}{3} + C$

So, option (c) is correct.

5. We have, $\frac{dy}{dx} = y \sin 2x$

$$\Rightarrow \quad \int \frac{dy}{y} = \int \sin 2x\, dx \Rightarrow \log|y| = -\frac{\cos 2x}{2} + C$$

Since $x = 0, y = 1$, therefore $C = 1/2$

Now, $\log|y| = \frac{1}{2}(1 - \cos 2x)$

$$\Rightarrow \quad \log|y| = \sin^2 x \Rightarrow y = e^{\sin^2 x}$$

So, option (a) is correct.

Case Study 5

A thermometer reading 0°F is taken outside. Ten minutes later, the thermometer reads 70°F. After another 5 min, the thermometer reads 50°F. At any time t, the thermometer reading be T°F and the outside temperature be S°F.

Based on the above information, solve the following questions:

Q 1. If λ is positive constant of proportionality, then find $\frac{dT}{dt}$.

Q 2. Find the general solution of differential equation formed in given situation.

Q 3. Find the value of constant of integration C in the solution of differential equation formed in given situation.

Or

15 minutes later, the thermometer reads 50°F, find the outside temperature.

Solutions

1. Given, at any time t, the thermometer reading be $T°$F and the outside temperature be $S°$F.

   Then, by Newton's law of cooling, we have

$$\frac{dT}{dt} \propto (T-S) \Rightarrow \frac{dT}{dt} = -\lambda(T-S)$$

2. We have, $\frac{dT}{dt} = -\lambda(T-S)$

$$\Rightarrow \quad \frac{dT}{T-S} = -\lambda\, dt$$

$$\Rightarrow \quad \int \frac{1}{T-S}\, dT = -\lambda \int dt$$

$$\Rightarrow \quad \log|T-S| = -\lambda t + C$$

3. Since, at $t = 0$, $T = 100°$F

$$\therefore \quad \log|100-S| = 0 + C$$

$$\Rightarrow \quad C = \log|100-S|$$

Or

Since, at $t = 15$, $T = 50°$F

$$\therefore \quad \log|50-S| = -\lambda \times 15 + \log|100-S|$$

$$\Rightarrow \quad 15\lambda = \log\left|\frac{100-S}{50-S}\right|$$

$$\Rightarrow \quad e^{15\lambda} = \frac{100-S}{50-S} \Rightarrow S = 50\left(\frac{2-e^{15\lambda}}{1-e^{15\lambda}}\right)$$

Case Study 6

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum.

Based on the given information, solve the following questions:

Q 1. If P_0 is the initial principal, then find the solution of differential equation formed in given situation.

Q 2. If the interest is compounded continuously at 5% per annum, in how many years will ₹ 100 double itself?

Or

At what interest rate will ₹ 100 double itself in 10 yr? ($\log_e 2 = 0.6931$)

Q 3. How much will ₹ 1000 be worth at 5% interest after 10 yr? ($e^{0.5} = 1.648$)

Solutions

1. Here, P denotes the principal at any time t and the rate of interest be $r\%$ per annum compounded continuously, then according to the law given in the problem, we get

$$\frac{dP}{dt} = \frac{Pr}{100}$$

$$\Rightarrow \quad \frac{dP}{P} = \frac{r}{100} dt \Rightarrow \int \frac{1}{P} dP = \frac{r}{100} \int dt$$

$$\Rightarrow \quad \log P = \frac{rt}{100} + C \qquad ...(1)$$

At $t = 0, P = P_0$

$$\therefore \quad C = \log P_0$$

So, $$\log P = \frac{rt}{100} + \log P_0$$

$$\Rightarrow \quad \log\left(\frac{P}{P_0}\right) = \frac{rt}{100} \qquad ...(2)$$

2. We have, $r = 5\%$, $P_0 = ₹\ 100$ and $P = ₹\ 200 = 2P_0$

Substituting these values in eq. (2), we get

$$\log 2 = \frac{5}{100} t$$

$$\Rightarrow \quad t = 20 \log_e 2 = 20 \times 0.6931 \text{ yr}$$

$$= 13.862 \text{ yr}$$

Or

We have,

$P_0 = ₹\ 100$, $P = ₹\ 200 = 2P_0$ and $t = 10$ yr

Substituting these values in eq. (2), we get

$$\log 2 = \frac{10r}{100}$$

$$\Rightarrow \quad r = 10 \log 2 = 10 \times 0.6931 = 6.931$$

3. We have, $P_0 = ₹\ 1000$, $r = 5\%$ and $t = 10$

Substituting these values in eq. (2), we get

$$\log\left(\frac{P}{1000}\right) = \frac{5 \times 10}{100} = \frac{1}{2} = 0.5$$

$$\Rightarrow \quad \frac{P}{1000} = e^{0.5} \Rightarrow P = 1000 \times 1.648 = ₹\ 1648$$

Case Study 7

An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous, if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogeneous function of degree n, if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above information, solve the following questions: (CBSE 2023)

Q 1. Show that $(x^2-y^2)dx+2xy\,dy=0$ is a differential equation of the type $\frac{dy}{dx}=g\left(\frac{y}{x}\right)$.

Q 2. Solve the above equation to find its general solution.

Solutions

1. We have,

$$(x^2-y^2)dx+2xy\,dy=0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y^2-x^2}{2xy}=F(x,\ y) \quad \text{(say)}$$

Now, $F(\lambda x,\ \lambda y)=\frac{\lambda^2y^2-\lambda^2x^2}{2\lambda x\,\lambda y}=\frac{\lambda^2(y^2-x^2)}{\lambda^2\,2xy}$

$$=\lambda^0\frac{(y^2-x^2)}{2xy}=\lambda^0\left\{\frac{\left(\frac{y}{x}\right)^2-1}{2\left(\frac{y}{x}\right)}\right\}$$

Here, degree of function is 0, so given differential equation is a homogeneous equation.

Hence, the given differential equation of the type $g\left(\frac{y}{x}\right)$.

2. $\because \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore \quad v+\frac{xdv}{dx}=\frac{v^2x^2-x^2}{2xvx}$$

$$\Rightarrow \quad \frac{xdv}{dx}=\frac{v^2-1}{2v}-v$$

$$\Rightarrow \quad \frac{xdv}{dx}=\frac{-v^2-1}{2v}$$

$$\Rightarrow \quad \frac{-2v}{v^2+1}dv=\frac{dx}{x}$$

Integrating both sides, we get

$$-\int\frac{2v}{v^2+1}dv=\int\frac{dx}{x}$$

$$\Rightarrow \quad -\log|v^2+1| = \log|x| + \log C$$

$$\Rightarrow \quad \log\left|\frac{1}{v^2+1}\right| = \log|x|C$$

$$\Rightarrow \quad \frac{1}{v^2+1} = xC$$

$$\Rightarrow \quad \frac{1}{\frac{y^2}{x^2}+1} = xC \qquad \left[\text{put } v = \frac{y}{x}\right]$$

$$\Rightarrow \quad \frac{x^2}{y^2+x^2} = xC \Rightarrow x = (x^2+y^2)C$$

Case Study 8

A first order and first degree differential equation in which the degree of dependent variable and its derivative is one and they do not get multiplied together, is called a linear differential equation.

Consider the given equation $\frac{dy}{dx} + Py = Q$. This equation is known as linear differential equation. Here integrating factor *i.e.*, $\text{IF} = e^{\int P\,dx}$ and the complete solution is given by

$$y\,(\text{IF}) = \int Q \times (\text{IF})\,dx + C,$$

where P and Q are constants or some function of x.
Now consider the given equation

$$dy = \cos x\,(2 - y\,\text{cosec}\,x)\,dx$$

Based on the above information, solve the following questions:

Q 1. Find the value of integrating factor (IF).

Q 2. Find the general solution of the given equation.

Q 3. If $y\left(\frac{\pi}{2}\right) = 2$, then find the particular solution of given equation.

Or

If $x = \frac{\pi}{6}$, then find the value of y.

Solutions

1. Given differential equation can be written in linear differential equation form

$$\frac{dy}{dx} = \cos x\,(2 - y\,\text{cosec}\,x)$$

$$\Rightarrow \quad \frac{dy}{dx} = 2\cos x - \cos x \cdot \text{cosec}\,x \cdot y$$

$$\Rightarrow \quad \frac{dy}{dx} + \cos x \cdot \frac{1}{\sin x} y = 2\cos x$$

$$\Rightarrow \quad \frac{dy}{dx} + \cot x \cdot y = 2\cos x$$

On comparing with $\frac{dy}{dx} + Py = Q$, we get

$$P = \cot x \text{ and } Q = 2\cos x$$

Now, $\text{IF} = e^{\int P\,dx} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$

2. Complete solution is

$$y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$$

$$\Rightarrow y \cdot \sin x = \int 2\cos x \cdot \sin x\,dx + C$$

$$\Rightarrow y \cdot \sin x = \int \sin 2x\,dx + C \Rightarrow y \cdot \sin x = -\frac{1}{2}\cos 2x + C$$

which is the required general solution.

3. Now, put $x = \frac{\pi}{2}$ and $y = 2$ in general solution, we get

$$2 \cdot \sin\frac{\pi}{2} = -\frac{1}{2}\cos 2 \cdot \frac{\pi}{2} + C \Rightarrow 2 \times 1 = -\frac{1}{2}\cos\pi + C$$

$$\Rightarrow 2 = -\frac{1}{2}(-1) + C \Rightarrow C = 2 - \frac{1}{2} = \frac{3}{2}$$

$\therefore$ Required particular solution is

$$y \cdot \sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$$

$$\Rightarrow \quad 2y\sin x = -\cos 2x + 3$$

Or

Now, put $x = \frac{\pi}{6}$ in the particular solution, we get

$$2y\sin\frac{\pi}{6} = -\cos 2 \cdot \frac{\pi}{6} + 3$$

$$\Rightarrow \quad 2y \times \frac{1}{2} = -\cos\frac{\pi}{3} + 3$$

$$\Rightarrow \quad y = -\frac{1}{2} + 3 = \frac{5}{2}$$

Solutions for Questions 9 to 18 are Given Below

Case Study 9

In a college hostel accommodating 1000 students, one of the hostellers came in carrying Corona virus, and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 50 infected students after 4 days.

Based on the above information, answer the following questions.

(i) If $n(t)$ denote the number of students infected by Corona virus at any time t, then maximum value of $n(t)$ is

(a) 50 (b) 100 (c) 500 (d) 1000

(ii) $\frac{dn}{dt}$ is proportional to

(a) $n(1000 - n)$ (b) $n(100 + n)$

(c) $n(100 - n)$ (d) $n(100 + n)$

(iii) The value of $n(4)$ is

(a) 1 (b) 50 (c) 100 (d) 1000

(iv) The most general solution of differential equation formed in given situation is

(a) $\frac{1}{1000}\log\left(\frac{1000-n}{n}\right) = \lambda t + c$ (b) $\log\left(\frac{n}{100-n}\right) = \lambda t + c$

(c) $\frac{1}{1000}\log\left(\frac{n}{1000-n}\right) = \lambda t + c$ (d) None of these

(v) The value of n at any time is given by

(a) $n(t)=\dfrac{1000}{1+999e^{-0.9906t}}$

(b) $n(t)=\dfrac{1000}{1-999e^{-0.9906t}}$

(c) $n(t)=\dfrac{100}{1-999e^{-0.996t}}$

(d) $n(t)=\dfrac{100}{999+e^{1000t}}$

Case Study 10

A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50 °F. At any time t the thermometer reading be T°F and the outside temperature be S °F.

Based on the above information, answer the following questions.

(i) If λ is positive constant of proportionality, then $\dfrac{dT}{dt}$ is

(a) $\lambda\,(T-S)$
(b) $\lambda\,(T+S)$
(c) λTS
(d) $-\lambda\,(T-S)$

(ii) The value of $T(5)$ is

(a) 30°F
(b) 40°F
(c) 50°F
(d) 60°F

(iii) The value of $T(10)$ is

(a) 50°F
(b) 60°F
(c) 80°F
(d) 90°F

(iv) Find the general solution of differential equation formed in given situation.

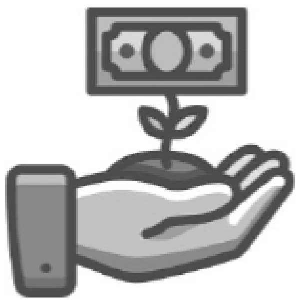
(a) $\log T = St + c$
(b) $\log (T-S) = -\lambda t + c$
(c) $\log S = tT + c$
(d) $\log (T+S) = \lambda t + c$

(v) Find the value of constant of integration c in the solution of differential equation formed in given situation.

(a) $\log (60-S)$
(b) $\log (80+S)$
(c) $\log (80-S)$
(d) $\log (60+S)$

Case Study 11

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum.



Based on the above information, answer the following questions.

(i) Find the value of $\dfrac{dP}{dt}$.

(a) $\dfrac{Pr}{1000}$
(b) $\dfrac{Pr}{100}$
(c) $\dfrac{Pr}{10}$
(d) Pr

(ii) If P_0 be the initial principal, then find the solution of differential equation formed in given situation.

(a) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$ (b) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{10}$ (c) $\log\left(\frac{P}{P_0}\right) = rt$ (d) $\log\left(\frac{P}{P_0}\right) = 100rt$

(iii) If the interest is compounded continuously at 5% per annum, in how many years will ₹ 100 double itself?

(a) 12.728 years (b) 14.789 years (c) 13.862 years (d) 15.872 years

(iv) At what interest rate will ₹ 100 double itself in 10 years? ($\log_e 2 = 0.6931$).

(a) 9.66% (b) 8.239% (c) 7.341% (d) 6.931%

(v) How much will ₹ 1000 be worth at 5% interest after 10 years? ($e^{0.5} = 1.648$).

(a) ₹ 1648 (b) ₹ 1500 (c) ₹ 1664 (d) ₹ 1572

Case Study 12

In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F, where the room temperature is 50°F. Also, it is given the body temperature at the time of death was normal, *i.e.*, 98.6°F.

Let T be the temperature of the body at any time t and initial time is taken to be 8 p.m.

Based on the above information, answer the following questions.

(i) By Newton's law of cooling, $\frac{dT}{dt}$ is proportional to

(a) $T - 60$ (b) $T - 50$ (c) $T - 70$ (d) $T - 98.6$

(ii) When $t = 0$, then body temperature is equal to

(a) 50°F (b) 60°F (c) 70°F (d) 98.6°F

(iii) When $t = 2$, then body temperature is equal to

(a) 50°F (b) 60°F (c) 70°F (d) 98.6°F

(iv) The value of T at any time t is

(a) $50+20\left(\frac{1}{2}\right)^{t}$ (b) $50+20\left(\frac{1}{2}\right)^{t-1}$ (c) $50+20\left(\frac{1}{2}\right)^{t/2}$ (d) None of these

(v) If it is given that $\log_e(2.43) = 0.88789$ and $\log_e(0.5) = -0.69315$, then the time at which the murder occur is

(a) 7:30 p.m. (b) 5:30 p.m. (c) 6:00 p.m. (d) 5:00 p.m.

Case Study 13

A rumour on whatsapp spreads in a population of 5000 people at a rate proportional to the product of the number of people who have heard it and the number of people who have not. Also, it is given that 100 people initiate the rumour and a total of 500 people know the rumour after 2 days.
Based on the above information, answer the following questions.

(i) If $y(t)$ denote the number of people who know the rumour at an instant t, then maximum value of $y(t)$ is

(a) 500 (b) 100 (c) 5000 (d) none of these

(ii) $\frac{dy}{dt}$ is proportional to

(a) $(y - 5000)$ (b) $y(y - 500)$ (c) $y(500 - y)$ (d) $y(5000 - y)$

(iii) The value of $y(0)$ is

(a) 100 (b) 500 (c) 600 (d) 200

(iv) The value of $y(2)$ is

(a) 100 (b) 500 (c) 600 (d) 200

(v) The value of y at any time t is given by

(a) $y = \frac{5000}{e^{-5000kt} + 1}$ (b) $y = \frac{5000}{1 + e^{5000kt}}$ (c) $y = \frac{5000}{49e^{-5000kt} + 1}$ (d) $y = \frac{5000}{49(1 + e^{-5000kt})}$

Case Study 14

Order : The order of a differential equation is the order of the highest order derivative appearing in the differential equation.
Degree : The degree of differential equation is the power of the highest order derivative, when differential coefficients are made free from radicals and fractions. Also, differential equation must be a polynomial equation in derivatives for the degree to be defined.
Based on the above information, answer the following questions.

(i) Find the degree of the differential equation $2\frac{d^2y}{dx^2} + 3\sqrt{1 - \left(\frac{dy}{dx}\right)^2} - y = 0$.

(a) 3 (b) 4 (c) 2 (d) 1

(ii) Order and degree of the differential equation $y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}$ are respectively

(a) 1, 1 (b) 1, 2 (c) 1, 3 (d) 1, 4

(iii) Find order and degree of the equation $y''' + y^2 + e^{y'} = 0$.

(a) order = 3, degree = undefined (b) order = 1, degree = 3
(c) order = 2, degree = undefined (d) order = 1, degree = 2

(iv) Determine degree of the differential equation $(\sqrt{a + x}) \cdot \left(\frac{dy}{dx}\right) + x = 0$

(a) 3 (b) not defined (c) 1 (d) 2

(v) Order and degree of the differential equation $\left(1+\left(\frac{dy}{dx}\right)^3\right)^{\frac{7}{3}} = 7\frac{d^2y}{dx^2}$ are respectively

(a) 2, 1 (b) 2, 3 (c) 1, 3 (d) $1, \frac{7}{3}$

Case Study 15

A differential equation is said to be in the variable separable form if it is expressible in the form $f(x)\,dx = g(y)\,dy$. The solution of this equation is given by

$\int f(x)dx = \int g(y)dy + c$, where c is the constant of integration.

Based on the above information, answer the following questions.

(i) If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of 'a' is

(a) 2 (b) – 2 (c) 3 (d) – 4

(ii) The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circle with

(a) variable radii and fixed centre (0, 1)
(b) variable radii and fixed centre (0, –1)
(c) fixed radius 1 and variable centre on x-axis
(d) fixed radius 1 and variable centre on y-axis

(iii) If $y' = y + 1$, $y(0) = 1$, then $y(\ln 2) =$

(a) 1 (b) 2 (c) 3 (d) 4

(iv) The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $e^x = \frac{y^3}{3} + e^y + c$ (b) $e^y = \frac{x^2}{3} + e^x + c$ (c) $e^y = \frac{x^3}{3} + e^x + c$ (d) none of these

(v) If $\frac{dy}{dx} = y \sin 2x$, $y(0) = 1$, then its solution is

(a) $y = e^{\sin^2 x}$ (b) $y = \sin^2 x$ (c) $y = \cos^2 x$ (d) $y = e^{\cos^2 x}$

Case Study 16

If an equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y\cdot(\text{I.F.}) = \int Q\cdot(\text{I.F.})\,dx + c$, where I.F. $= e^{\int P dx}$.

Now, suppose the given equation is $(1+\sin x)\frac{dy}{dx} + y\cos x + x = 0$.

Based on the above information, answer the following questions.

(i) The value of P and Q respectively are

(a) $\frac{\sin x}{1+\cos x}, \frac{x}{1+\sin x}$ (b) $\frac{\cos x}{1+\sin x}, \frac{-x}{1+\sin x}$ (c) $\frac{-\cos x}{1+\sin x}, \frac{x}{1+\sin x}$ (d) $\frac{\cos x}{1+\sin x}, \frac{x}{1+\sin x}$

(ii) The value of I.F. is

(a) $1 - \sin x$ (b) $\cos x$ (c) $1 + \sin x$ (d) $1 - \cos x$

(iii) Solution of given equation is

(a) $y(1 - \sin x) = x + c$ (b) $y(1 + \sin x) = -x^2 + c$

(c) $y(1 - \sin x) = \frac{-x^2}{2} + c$ (d) $y(1 + \sin x) = \frac{-x^2}{2} + c$

(iv) If $y(0) = 1$, then y equals

(a) $\frac{2-x^2}{2(1+\sin x)}$ (b) $\frac{2+x^2}{2(1+\sin x)}$ (c) $\frac{2-x^2}{2(1-\sin x)}$ (d) $\frac{2+x^2}{2(1-\sin x)}$

(v) Value of is $y\left(\frac{\pi}{2}\right)$ is

(a) $\frac{4-\pi^2}{2}$ (b) $\frac{8-\pi^2}{16}$ (c) $\frac{8-\pi^2}{4}$ (d) $\frac{4+\pi^2}{2}$

Case Study 17

If the equation is of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ or $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, so that the dependent variable y is changed to another variable v and then apply variable separable method.

Based on the above information, answer the following questions.

(i) The general solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is

(a) $\tan^{-1}\frac{x}{y} = \log|x| + c$ (b) $\tan^{-1}\frac{y}{x} = \log|x| + c$ (c) $y = x\log|x| + c$ (d) $x = y\log|y| + c$

(ii) Solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is

(a) $x^3 + y^2 = cx^2$ (b) $\frac{x^2}{2} + \frac{y^3}{3} = y^2 + c$ (c) $x^2 + y^3 = cx^2$ (d) $x^2 + y^2 = cx^3$

(iii) Solution of the differential equation $(x^2 + 3xy + y^2)dx - x^2\,dy = 0$ is

(a) $\frac{x+y}{x} - \log x = c$ (b) $\frac{x+y}{x} + \log x = c$ (c) $\frac{x}{x+y} - \log x = c$ (d) $\frac{x}{x+y} + \log x = c$

(iv) General solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$ is

(a) $\log(xy) = c$ (b) $\log y = cx$ (c) $\log\left(\frac{y}{x}\right) = cx$ (d) $\log x = cy$

(v) Solution of the differential equation $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = x^2\cos x$ is

(a) $e^{\frac{y}{x}} - \sin x = c$ (b) $e^{\frac{y}{x}} + \sin x = c$ (c) $e^{\frac{-y}{x}} - \sin x = c$ (d) $e^{\frac{-y}{x}} + \sin x = c$

Case Study 18

If the equation is of the form $\frac{dy}{dx}+Py=Q$, where P, Q are functions of x, then the solution of the differential equation is given by $ye^{\int P dx}=\int Qe^{\int P dx}dx+c$, where $e^{\int P dx}$ is called the integrating factor (I.F.).

Based on the above information, answer the following questions.

(i) The integrating factor of the differential equation $\sin x\frac{dy}{dx}+2y\cos x=1$ is $(\sin x)^{\lambda}$, where $\lambda=$

(a) 0 (b) 1 (c) 2 (d) 3

(ii) Integrating factor of the differential equation $(1-x^2)\frac{dy}{dx}-xy=1$ is

(a) $-x$ (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}\log(1-x^2)$

(iii) The solution of $\frac{dy}{dx}+y=e^{-x}$, $y(0)=0$, is

(a) $y=e^x(x-1)$ (b) $y=xe^{-x}$ (c) $y=xe^{-x}+1$ (d) $y=(x+1)e^{-x}$

(iv) General solution of $\frac{dy}{dx}+y\tan x=\sec x$ is

(a) $y\sec x=\tan x+c$ (b) $y\tan x=\sec x+c$ (c) $\tan x=y\tan x+c$ (d) $x\sec x=\tan y+c$

(v) The integrating factor of differential equation $\frac{dy}{dx}-3y=\sin 2x$ is

(a) e^{3x} (b) e^{-2x} (c) e^{-3x} (d) xe^{-3x}

HINTS & EXPLANATIONS

9. (i) (d): Since, maximum number of students in hostel is 1000.

$\therefore$ Maximum value of $n(t)$ is 1000.

(ii) (a): Clearly, according to given information, $\frac{dn}{dt}=\lambda n(1000-n)$, where λ is constant of proportionality.

(iii) (b): Since, 50 students are infected after 4 days.

$\therefore$ $n(4)=50$.

(iv) (c): We have, $\frac{dn}{dt}=\lambda n(100-n)$

$$\Rightarrow \int\frac{dn}{n(1000-n)}=\lambda\int dt$$

$$\Rightarrow \frac{1}{1000}\int\left(\frac{1}{1000-n}+\frac{1}{n}\right)dn=\lambda\int dt$$

$$\Rightarrow \frac{1}{1000}\left[\frac{\log(1000-n)}{-1}+\log n\right]=\lambda t+C$$

$$\Rightarrow \frac{1}{1000}\log\left(\frac{n}{1000-n}\right)=\lambda t+C$$

(v) (a): When, $t=0$, $n=1$

This condition is satisfied by option (a) only.

10. (i) (d): Given, at any time t the thermometer reading be T°F and the outside temperature be S°F. Then, by Newton's law of cooling, we have

$$\frac{dT}{dt}\propto(T-S) \Rightarrow \frac{dT}{dt}=-\lambda(T-S)$$

(ii) (d): Since, after 5 minutes, thermometer reads 60°F

$\therefore$ Value of $T(5)=60$°F

(iii) (a): Clearly from given information, value of $T(10)$ is 50°F.

(iv) (b): We have, $\frac{dT}{dt}=-\lambda(T-S)$

$\Rightarrow \dfrac{dT}{T-S} = -\lambda\, dt \Rightarrow \int \dfrac{1}{T-S} dT = -\lambda \int dt$

$\Rightarrow \log(T-S) = -\lambda t + c$

(v) (c) : Since, at $t = 0$, $T = 80°F$

$\therefore \log(80 - S) = 0 + c \Rightarrow c = \log(80 - S)$

11. (i) (b): Here, P denotes the principal at any time t and the rate of interest be r% per annum compounded continuously, then according to the law given in the problem, we get

$$\frac{dP}{dt} = \frac{Pr}{100}$$

(ii) (a): We have, $\dfrac{dP}{dt} = \dfrac{Pr}{100}$

$\Rightarrow \dfrac{dP}{P} = \dfrac{r}{100} dt \Rightarrow \int \dfrac{1}{P} dP = \dfrac{r}{100} \int dt$

$\Rightarrow \log P = \dfrac{rt}{100} + C$ ...(1)

At $t = 0$, $P = P_0$.

$\therefore C = \log P_0$

So, $\log P = \dfrac{rt}{100} + \log P_0$

$\Rightarrow \log\left(\dfrac{P}{P_0}\right) = \dfrac{rt}{100}$ ...(2)

(iii) (c) : We have, $r = 5$, $P_0 = ₹\,100$ and $P = ₹\,200 = 2P_0$

Substituting these values in (2), we get

$\log 2 = \dfrac{5}{100} t$

$\Rightarrow t = 20 \log_e 2 = 20 \times 0.6931$ years $= 13.862$ years

(iv) (d): We have,

$P_0 = ₹\,100$, $P = ₹\,200 = 2P_0$ and $t = 10$ years

Substituting these values in (2), we get

$\log 2 = \dfrac{10r}{100} \Rightarrow r = 10 \log 2 = 10 \times 0.6931 = 6.931$

(v) (a): We have

$P_0 = ₹\,1000$, $r = 5$ and $t = 10$

Substituting these values in (2), we get

$$\log\left(\frac{P}{1000}\right) = \frac{5\times 10}{100} = \frac{1}{2} = 0.5 \Rightarrow \frac{P}{1000} = e^{0.5}$$

$\Rightarrow P = 1000 \times 1.648 = ₹\,1648$

12. (i) (b): Given, T is the temperature of the body at any time t. Then, by Newton's law of cooling, we get

$\dfrac{dT}{dt} = k(T - 50)$, where k is the constant of proportionality.

(ii) (c) : From given information, we have

At 8 p.m. temperature is 70°F

$\therefore$ At $t = 0$, $T = 70°F$

(iii) (b): From given information, we have

At 10 p.m., temperature is 60°F

$\therefore$ At $t = 2$, $T = 60°$ F

(iv) (c) : $\dfrac{dT}{dt} = k(T-50) \Rightarrow \dfrac{dT}{T-50} = k\, dt$

On integrating both sides, we get

$\log|T - 50| = kt + \log C \Rightarrow T - 50 = Ce^{kt}$

Clearly, for $t = 0$, $T = 70° \Rightarrow C = 20$

Thus, $T - 50 = 20e^{kt}$

For $t = 2$, $T = 60° \Rightarrow 10 = 20\, e^{2k}$

$\Rightarrow 2k = \log\left(\dfrac{1}{2}\right) \Rightarrow k = \dfrac{1}{2}\log\left(\dfrac{1}{2}\right)$

Hence, $T = 50 + 20\left(\dfrac{1}{2}\right)^{\frac{t}{2}}$

(v) (b): We have, $T = 50 + 20\left(\dfrac{1}{2}\right)^{t/2}$

Now, $98.6 = 50 + 20\left(\dfrac{1}{2}\right)^{\frac{t}{2}}$

$\Rightarrow \dfrac{48.6}{20} = \left(\dfrac{1}{2}\right)^{\frac{t}{2}} \Rightarrow \log\left(\dfrac{48.6}{20}\right) = \dfrac{t}{2}\log\left(\dfrac{1}{2}\right)$

$$\Rightarrow \frac{t}{2} = \frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)} \Rightarrow t = 2\left(\frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)}\right) \approx -2.56$$

So, it appears that the person was murdered 2.5 hours before 8 p.m. *i.e.*, about 5 : 30 p.m.

13. (i) (c) : Since, size of population is 5000.

$\therefore$ Maximum value of $y(t)$ is 5000.

(ii) (d): Clearly, according to given information,

$\dfrac{dy}{dt} = ky(5000 - y)$, where k is the constant of proportionality.

(iii) (a) : Since, rumour is initiated with 100 people.

$\therefore$ When $t = 0$, then $y = 100$

Thus $y(0) = 100$

(iv) (b): Since, rumour is spread in 500 people, after 2 days.

$\therefore$ When $t = 2$, then $y = 500$.

Thus, $y(2) = 500$

(v) (c) : We know that, when $t = 0$, then $y = 100$

This condition is satisfied by option (c) only.

14. (i) (c) : We have, $2\dfrac{d^2y}{dx^2} + 3\sqrt{1 - \left(\dfrac{dy}{dx}\right)^2} - y = 0$

$\therefore 2\dfrac{d^2y}{dx^2} = -3\sqrt{1 - \left(\dfrac{dy}{dx}\right)^2} - y$

Squaring both sides, we get

$$4\left(\frac{d^2y}{dx^2}\right)^2 = 9\left[1-\left(\frac{dy}{dx}\right)^2 - y\right]$$

Here, highest order derivative is $\frac{d^2y}{dx^2}$ and its power is 2. So, its degree is 2.

(ii) (d): We have, $y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}$

$\Rightarrow \quad y\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right)^4 = x$

$\Rightarrow$ Here, highest order derivative is $\frac{dy}{dx}$. So, its order is 1 and degree is 4.

(iii) (a): We have, $y''' + y^2 + e^{y'} = 0$

$$\frac{d^3y}{dx^3} + y^2 + e^{(dy/dx)} = 0$$

Highest order derivative is $\frac{d^3y}{dx^3}$. So, its order is 3.

Also, the given differential cannot be expressed as a polynomial. So, its degree is not defined.

(iv) (c): The given differential equation is,

$$\sqrt{a+x}\cdot\left(\frac{dy}{dx}\right) + x = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{a+x}}$$

Clearly, degree = 1.

(v) (b): We have $\left(1+\left(\frac{dy}{dx}\right)^3\right)^{\frac{7}{3}} = 7\frac{d^2y}{dx^2}$

$\Rightarrow \left(1+\left(\frac{dy}{dx}\right)^3\right)^7 = \left(7\frac{d^2y}{dx^2}\right)^3$

$\therefore$ Order is 2 and degree is 3.

15. (i) (b): We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$\Rightarrow (ax+3)dx = (2y+f)dy$

$\Rightarrow a\frac{x^2}{2} + 3x = y^2 + fy + c$ (Integrating)

$\Rightarrow -\frac{a}{2}x^2 + y^2 - 3x + fy + C = 0$

This will represent a circle, if $\frac{-a}{2} = 1 \Rightarrow a = -2$

[$\because$ In circle, coefficient of x^2 = coefficient of y^2]

(ii) (c): We have, $\frac{ydy}{\sqrt{1-y^2}} = dx$

On integration, we get $-\sqrt{1-y^2} = x + c$

$\Rightarrow 1-y^2 = (x+c)^2 \Rightarrow (x+c)^2 + y^2 = 1$, which represents a circle with radius 1 and centre on the x-axis.

(iii) (c): $y' = y+1 \Rightarrow \frac{dy}{y+1} = dx$

$\Rightarrow \ln(y+1) = x + c$ (Integrating)

Now, $y(0) = 1 \Rightarrow c = \ln 2$

$\therefore \ln\left(\frac{y+1}{2}\right) = x \Rightarrow y + 1 = 2e^x$

So, $y(\ln 2) = -1 + 2e^{\ln 2} = -1 + 4 = 3$

(iv) (c): From the given differential equation, we have

$$\frac{dy}{dx} = \frac{e^x + x^2}{e^y} \Rightarrow e^y dy = (e^x + x^2)\,dx$$

Integrating, we get $e^y = e^x + \frac{x^3}{3} + c$

(v) (a): We have, $\frac{dy}{dx} = y\sin 2x$

$\Rightarrow \frac{dy}{y} = \sin 2x\,dx \Rightarrow \log y = -\frac{\cos 2x}{2} + c$

Since $x = 0, y = 1$ therefore $c = 1/2$

Now, $\log y = \frac{1}{2}(1-\cos 2x)$

$\Rightarrow \log y = \sin^2 x \Rightarrow y = e^{\sin^2 x}$

16. (i) (b): The given differential equation can be written as $\frac{dy}{dx} + \frac{\cos x}{1+\sin x}y = \frac{-x}{1+\sin x}$

Compare it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{\cos x}{1+\sin x} \text{ and } Q = \frac{-x}{1+\sin x}$$

(ii) (c): I.F. $= e^{\int P dx} = e^{\int \frac{\cos x}{1+\sin x}dx}$

Put $1 + \sin x = t \Rightarrow \cos x\,dx = dt$

$\therefore$ I.F. $= e^{\int \frac{1}{t}dt} = e^{\log t} = t = 1 + \sin x$

(iii) (d): Solution of given differential equation is given by $y.(\text{I.F.}) = \int Q\,(\text{I.F.})\,dx + c$

$\Rightarrow y(1+\sin x) = \int \frac{-x}{1+\sin x}.(1+\sin x)\,dx + c$

$\Rightarrow y(1+\sin x) = \frac{-x^2}{2} + c$

(iv) (a): We have, $y(0) = 1$ *i.e.*, $x = 0, y = 1$

$\therefore 1(1+\sin 0) = c \Rightarrow c = 1$

$\therefore y(1+\sin x) = \frac{-x^2}{2} + 1 = \frac{2-x^2}{2}$

$\therefore y = \frac{2-x^2}{2(1+\sin x)}$

(v) (b): We have, $y = \frac{2-x^2}{2(1+\sin x)}$

$\therefore \quad y\left(\frac{\pi}{2}\right)=\frac{2-\left(\frac{\pi}{2}\right)^2}{2\left(1+\sin\frac{\pi}{2}\right)}=\frac{2-\frac{\pi^2}{4}}{4}=\frac{8-\pi^2}{16}$

17. (i) (b): We have, $\frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$

Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \quad v+x\frac{dv}{dx}=\frac{x^2+x\cdot vx+v^2x^2}{x^2}=1+v+v^2$

$\Rightarrow \quad x\frac{dv}{dx}=1+v^2 \Rightarrow \int\frac{dv}{1+v^2}=\int\frac{dx}{x}+c$

$\Rightarrow \quad \tan^{-1}v=\log|x|+c \Rightarrow \tan^{-1}\frac{y}{x}=\log|x|+c$

(ii) (d): We have,

$2xy\frac{dy}{dx}=x^2+3y^2 \Rightarrow \frac{dy}{dx}=\frac{x^2+3y^2}{2xy}$

Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \quad v+x\frac{dv}{dx}=\frac{x^2+3v^2x^2}{2vx^2} \Rightarrow x\frac{dv}{dx}=\frac{1+3v^2}{2v}-v$

$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v^2}{2v} \Rightarrow \int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}+\log c$

$\Rightarrow \quad \log|1+v^2|=\log|x|+\log|c| \Rightarrow \log|v^2+1|=\log|xc|$

$\Rightarrow \quad v^2+1=xc \Rightarrow \frac{y^2}{x^2}+1=xc \Rightarrow x^2+y^2=x^3c$

(iii) (d): We have, $(x^2+3xy+y^2)\,dx-x^2\,dy=0$

$\Rightarrow \quad \frac{x^2+3xy+y^2}{x^2}=\frac{dy}{dx}$

Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \quad \frac{x^2+3x^2v+x^2v^2}{x^2}=\left(v+x\frac{dv}{dx}\right)$

$\Rightarrow \quad 1+3v+v^2=v+x\frac{dv}{dx} \Rightarrow 1+2v+v^2=x\frac{dv}{dx}$

$\Rightarrow \quad \int\frac{dx}{x}-\int(v+1)^{-2}dv=c \Rightarrow \log x+\frac{1}{v+1}=c$

$\Rightarrow \quad \log x+\frac{x}{x+y}=c$

(iv) (c): We have, $\frac{dy}{dx}=\frac{y}{x}\left\{\log\left(\frac{y}{x}\right)+1\right\}$

Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \quad v+x\frac{dv}{dx}=v\{\log(v)+1\} \Rightarrow x\frac{dv}{dx}=v\log v$

$\Rightarrow \quad \int\frac{dv}{v\log v}=\int\frac{dx}{x} \Rightarrow \log|\log v|=\log|x|+\log|c|$

$\Rightarrow \quad \log\left(\frac{y}{x}\right)=cx$

(v) (a): We have, $\left(x\frac{dy}{dx}-y\right)e^{\frac{y}{x}}=x^2\cos x$

$\Rightarrow \quad \left(\frac{dy}{dx}-\frac{y}{x}\right)e^{\frac{y}{x}}=x\cos x$

Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow \quad \left(v+x\frac{dv}{dx}-v\right)e^v=x\cos x \Rightarrow xe^v\frac{dv}{dx}=x\cos x$

$\Rightarrow \quad \int e^v dv=\int\cos x\,dx \Rightarrow e^v=\sin x+c$

$\Rightarrow \quad e^{\frac{y}{x}}-\sin x=c$

18. (i) (c): The given differential equation can be written as $\frac{dy}{dx}+2y\cot x=\operatorname{cosec}x$

$\therefore \quad$ I.F. $=e^{\int 2\cot x\,dx}=e^{2\log|\sin x|}=(\sin x)^2$

$\therefore \quad \lambda=2$

(ii) (c): We have, $(1-x^2)\frac{dy}{dx}-xy=1$

$\Rightarrow \quad \frac{dy}{dx}-\frac{x}{1-x^2}\cdot y=\frac{1}{1-x^2}$

$\therefore \quad$ I.F. $=e^{-\int\frac{x}{1-x^2}dx}=e^{\frac{1}{2}\int\frac{-2x}{1-x^2}dx}$

$=e^{\frac{1}{2}\log(1-x^2)}=e^{\log(1-x^2)^{\frac{1}{2}}}=\sqrt{1-x^2}$

(iii) (b): We have, $\frac{dy}{dx}+y=e^{-x}$

It is a linear differential equation with I.F. $=e^{\int dx}=e^x$

Now, solution is $y\cdot e^x=\int e^x\cdot e^{-x}\,dx+c$

$\Rightarrow \quad ye^x=\int dx+c \Rightarrow ye^x=x+c \Rightarrow y=xe^{-x}+ce^{-x}$

$\because \quad y(0)=0 \Rightarrow c=0 \therefore y=xe^{-x}$

(iv) (a): We have, $\frac{dy}{dx}+y\tan x=\sec x$

It is a linear differential equation with

I.F. $=e^{\int\tan x dx}=e^{\log|\sec x|}=\sec x$

Now, solution is $y\sec x=\int\sec^2 x\,dx+c$

$\Rightarrow \quad y\sec x=\tan x+c$

(v) (c): We have, $\frac{dy}{dx}-3y=\sin 2x$

It is a linear differential equation with

I.F. $=e^{\int -3dx}=e^{-3x}$